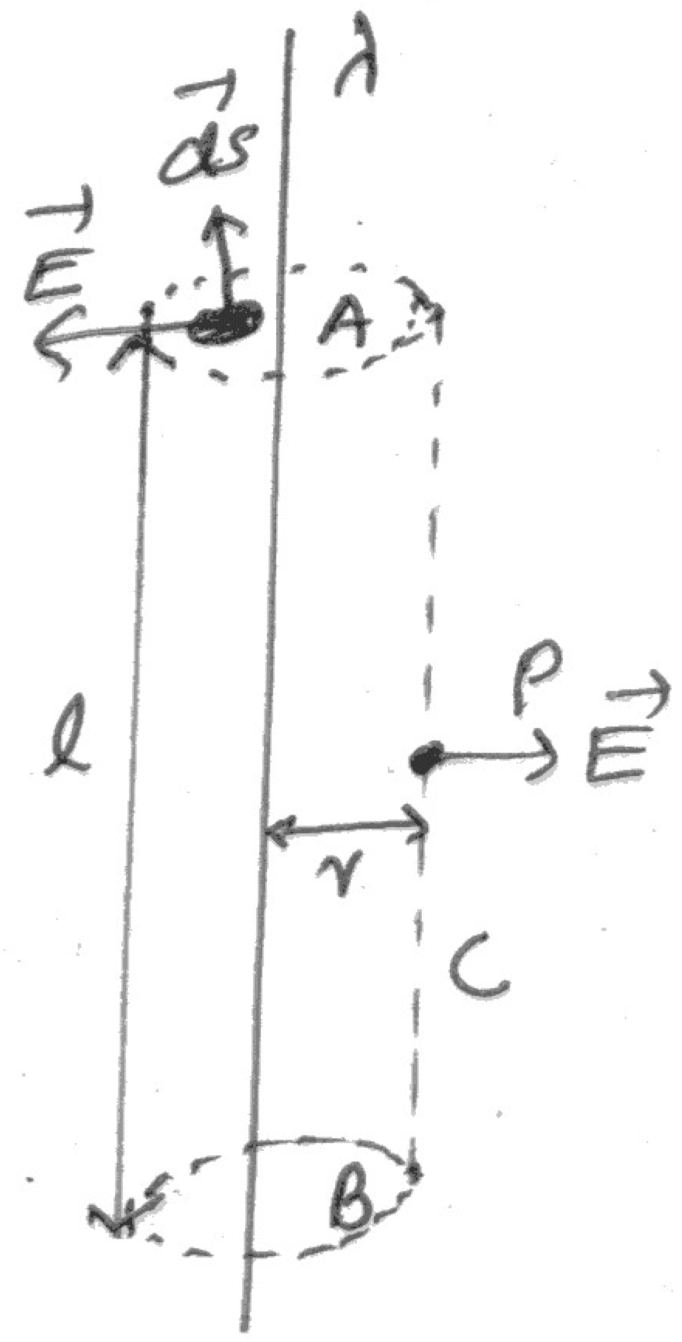
Electric field due to an infinite line of charge :- Let there be a wire of infinite length having a constant line charge density λ. Let P be a point at a distance r from the straight line charge.

The direction of the electric field due to the this electric charge will be radially outward due to symmetry. Draw a cylinder of height 'l' and radius 'r' coaxial with the line charge and closed at both ends by plain caps A and B normal to the axis to represent the Gaussian surface. The closed surface consists of three parts A, B and C. Now we consider a very small area $\vec{ds}$ on the surface A, then area vector $\vec{ds}$ the outward drawn normal acts along the outward drawn normal and is at right angles to the electric field vector $\vec{E}$.



$\therefore$ For the surface A; $\iint_A \vec{E}.\vec{ds} = 0$

Similarly for the surface B ; $\iint_B \vec{E}.\vec{ds} = 0$

For the surface c the area vector $\vec{ds}$ and the electric field vector $\vec{E}$ are parallel and act in the same direction.

$$\iint_c \vec{E}.\vec{ds} = \iint_c E, ds$$

where E and ds are the magnitude of vector $\vec{E}$ and $\vec{ds}$ respectively.

As all point on the curved surface of the cylinder c are equidistant from the axis, the electric field is the same at every point. Hence.

$$E = \text{a constant}$$

The area of curved surface is $2\pi r l$.

Hence the electric flux through the curved surface of Gaussian surface is

$$\phi = \iint_c \vec{E}.\vec{ds} = \iint_c E\,dS = E \iint_c ds = E\,2\pi r l$$

According to Gauss's law, the electric flux through the Gaussian surface

$$\phi = \frac{1}{\epsilon_0} \lambda l$$

... the Gaussian surface = charge on length l of the

$\therefore \quad E \cdot 2\pi r l = \frac{1}{\epsilon_0} \lambda l$

or $\quad \boxed{E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}}$

The direction of $\vec{E}$ is perpendicular to the charged wire at every point and inversely proportional to the distance 'r' from the wire.

Electric potential at point P is

$$V = -\int E\,dr + \text{constant}$$

or $$V = -\int \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}\,dr + \text{constant}$$

$$\boxed{V = -\frac{\lambda}{2\pi\epsilon_0} \log_e r + \text{Constant}}$$